REVIEW

by Prof. Dr. Todor Dimitrov Todorov, Technical University - Gabrovo, of the documents submitted for participation in a competition for the academic position of "professor" in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics, scientific specialty "Computational Mathematics" announced in the State Gazette, issue 48, June 13, 2025. In the competition for a professor, announced in the State Gazette, issue 48, June 13, 2025, and on the website of TU-Gabrovo for the needs of the Department of "Mathematics, Informatics and Natural Sciences" at the Faculty of "Economics", the only candidate participating is Assoc. Prof. DSc Milena Radoslavova Racheva, Department of "Mathematics, Informatics and Natural Sciences" at the Faculty of "Economics" of the Technical University - Gabrovo.

1. Brief biographical information about the candidate

Assoc. Prof. Dr. Milena Radoslavova Racheva began her scientific practice as an Assistant Professor at the Polytechnic Institute of Mechanical and Electrical Engineering - Lovech in 1992. Since 1994, she has been working at the Technical University - Gabrovo as an Assistant Professor in the Department of Mathematics. From 1999 to 2003, Assoc. Prof. Racheva was a part-time PhD student with a scientific supervisor, Prof. DSc Andrey Andreev. The PhD study is conducted at the Institute of Parallel Information Processing - BAS, Sofia. In 2003, the candidate received the PhD degree from the SSC of Informatics and Applied Mathematics, HAC, Sofia, in the major 01.01.09. "Computational Mathematics". After receiving the PhD degree, Assoc. Prof. Racheva carried out a fruitful specialization in Gothenburg, Sweden. In 2006, Assoc. Prof. Racheva was elected as an Associate Professor in the Department of Mathematics of the Technical University of Gabrovo. In 2014, the Institute of Information and Communication Technologies - BAS, Sofia awarded Assoc. Prof. Racheva the scientific degree "Doctor of Science" in Mathematics (01.01.09. Computational Mathematics). Since 2022, the candidate has been the editor-in-chief of the international journal "Mathematical Modeling", Print ISSN 2535-0986, Online ISSN 2603-2929.

2. General description of the documents presented

Thirty-seven scientific publications in journals and conference proceedings have been submitted under this competition, of which 36 have already been published, and one publication has been approved for publication. The candidate submits only 5 independent publications. The submitted scientific publications are ranked as follows:

- three papers in journals referenced in Web of Science;
- twelve papers in non-refereed journals excluding Journal of the Technical University of Gabrovo;
- two papers in Journal of the Technical University of Gabrovo;
- one report at a conference abroad;

- 15 reports at conferences in Bulgaria without UniTech;
- four reports at the conference of TU-Gabrovo UniTech.

Taking into account the fact that some of the conference reports in Bulgaria are printed in the Springer series, we get:

- one publication in a journal referenced in quartile 1 of Web of Science;
- eight publications in journals referenced in quartile 4 of Web of Science (One of these publications has only been accepted for publication and is not currently available in Web of Science);
- six publications in journals that are refereed in Scopus but not in Web of Science;
- ten articles in refereed journals, excluding Journal of the Technical University of Gabrovo. Six of these papers have been published in the journal Mathematical Modeling, where Assoc. Prof. Racheva serves as the editor-in-chief;
- two papers in Journal of the Technical University of Gabrovo;
- reports in conference proceedings which are not referenced in Web of Science or Scopus.

The candidate declares participation in 17 projects, in four of which she has been the leader. Five of the projects are national. Four of the national projects were before Assoc. Prof. Racheva graduated as a doctor of science, and the fifth began in 2025 and has not been completed. Assoc. Prof. Racheva is the author of one textbook and co-author of another with respect to this procedure.

The presented documents meet the requirements, conditions, and criteria of the Law on the Development of the Academic Staff in the Republic of Bulgaria, rules for applying the mentioned above law, rules for the conditions and order for acquiring academic degrees and academic positions at Technical University - Gabrovo to occupy the academic position “Professor”.

3. Impact of the candidate's scientific publications in the scientific community (known citations)

Ninety-six citations from reputable journals are presented.

4. Overview of the content and results in the scientific works submitted

Assoc. Prof. Racheva's publications are in a large number of different fields. I classify the publications in mathematics into six scientific works.

4.1 Publications by the candidate with results that I rate the highest

Of the scientific papers written by Assoc. Prof. Racheva, for this competition, I rate the results in publications [8.5] and [8.6] the highest.

In the conference paper [8.5], an infinite element method is developed for solving second-order elliptic spectral problems in an infinite domain. A conforming coupling between finite and infinite elements is implemented. Original

basis functions that guarantee continuity of the test functions from the space $\boldsymbol{V_h}$ are obtained.

Article [8.6] is undoubtedly the greatest success of Assoc. Prof. Racheva. This article was published in a journal that is referenced in quartile 1 of Web of Science. The authors apply a discontinuous Galerkin method to solve an integro-differential equation with a weak singular convolution kernel, including fractional derivatives. This integro-differential equation models dynamic viscoelasticity of fractional order.

4.2 Conforming and non-conforming methods for solving spectral problems

In publications [8.7], [8.8], [8.18], [8.20], and [8.26], Assoc. Prof. Racheva concentrates on the study of nonconforming finite element approximations, finite elements with integral degrees of freedom, and two-sided estimates for eigenvalues of differential operators.

Conforming methods for estimating the eigenvalues of differential operators are applied in publications [8.9], [8.10], [8.11], [8.13], [8.19], [8.21], [8.25] and [8.26]. Using conforming methods for solving spectral problems, it is easy to obtain an estimate of the eigenvalues from above. In this scientific work, new methods for estimating the eigenvalues from below are developed. Article [8.10] is devoted to an original method for accelerating convergence. Ultraconvergence is proven for finite element approximations of the solution of the biharmonic problem with zero boundary conditions.

4.3 Two-level methods for solving isoparametric problems

This scientific work is based on publications [8.16] and [8.17]. I do not comment on these publications as I am a co-author on them.

4.4 Computational approaches for solving boundary value problems for ordinary differential equations and related spectral problems

In this scientific work, the candidate has developed methods for linearizing a quadratic Sturm-Liouville problem of the second and fourth orders, and an error estimate has been provided. A finite element method is developed for spectral problems of the sixth order. Without belittling the candidate's achievements, I would like to clarify. In publications [8.1], [8.2], [8.3], [8.4], [8.14], [8.27], boundary value problems or eigenvalue problems in one-dimensional domains are considered. The significant advantages of the finite element method cannot be demonstrated on boundary value problems for solving ordinary differential equations or eigenvalue problems in one-dimensional domains. When dividing an interval, we can always easily construct sequences of hierarchical uniform triangulations. This is obviously impossible for two-dimensional domains with complex geometry. The generation of uniform meshes significantly facilitates the solution finding of ordinary differential equations by the finite element method. When solving such boundary value and eigenvalue problems, the finite element

method has serious competitors, such as Runge and Kutta methods, the finite difference method, predictor-corrector methods, etc.

4.5 Applications of trigonometric polynomials and Fourier transforms

In this scientific work, the candidate presents applications of the Fourier transform in signal theory. Efficient methods for calculating improper integrals have been developed. Discrete approximations for solving the original and inverse problems for periodic functions have been obtained.

4.6 Creating applied mathematical models

This scientific work deals with mathematical modeling of hydraulic and mechanical processes. Real engineering problems are modeled by means of second-order hyperbolic equations. In the article [8.22], the authors present an original method for solving a second-order hyperbolic partial differential equation.

Publications outside mathematics

Publications [8.30] and [8.31] are related to the results of methodological work and the use of various types of software. They do not contain new results in mathematics, so I do not include them to any of the listed scientific works.

Publications [8.12], [8.15], [8.28], [8.29], [8.32], [8.33], [8.34] are in the field of engineering. They do not contain new results in the field of mathematics, so I consider them as additional publications for this competition.

5. General characteristics of the candidate's activities

5.1. Teaching and methodological activities

Assoc. Prof. Racheva is a lecturer with many years of experience in teaching mathematics at both the bachelor's and master's levels.

5.2. Scientific and applied science activities

Assoc. Prof. Milena Radoslavova Racheva is the editor-in-chief of the international journal Mathematical Modeling. She successfully participates in national and university scientific projects, some of which are in the area of applied sciences.

6. Contributions

Assoc. Prof. Racheva declares a wide range of scientific and applied science contributions. I will focus only on the essential part of them, which represent significant mathematical results.

- An infinite element method is presented for solving differential equations in an infinite domain. A conforming coupling of finite and infinite elements is implemented.
- An estimate of the error in the finite element approximations of the solution of a fractional order integro-differential equation with a weak singular kernel has been made. Such equations are used in viscoelasticity modeling.

- New numerical methods for solving boundary value and eigenvalue problems have been developed. The convergence of finite element approximations has been accelerated by using post processing methods. Higher order differential equations (fourth and sixth order) have been solved.
- Two-sided estimates have been made for the eigenvalues of differential operators.
- An analysis of nonconforming methods for solving eigenvalue problems has been made. An important theorem for identity finite element spaces has been proved.

7. Assessment of the candidate's personal contribution

All significant publications in mathematics without [8.6] were obtained in co-authorship with Prof. Dr. Andrey Andreev, who is the scientific supervisor of Assoc. Prof. Racheva. Article [8.6] was written with two co-authors, one of whom is a world-famous scientist. Assoc. Prof. Racheva successfully participates in scientific teams.

8. Critical notes and recommendations

8.1 Reports [8.4] and [8.14] contain an identical analysis of the Sturm-Liouville problem of order six. Equation (4) from report [8.14] is actually (8) from report [8.4]. Equation (8) from paper [8.4] should have been cited in report [8.14] without deriving the same result again.

8.2 In the paper [Figure 1a) and 1b), 8.26] elements with integral degrees of freedom are considered. In English, they are incorrectly called “8-point” and “9-point”. Since 4 (5) degrees of freedom are defined respectively along the sides of the element (along the sides of the element and along the entire element), the elements are “4-node” (“4-node”). Here, I emphasize that integral degrees of freedom do not have local properties.

8.3 In the paper [8.10], an original method for accelerating the convergence of finite element approximations to the eigenpairs of a biharmonic operator in a polyrectangular domain is developed. I have the following critical remarks about this publication.

- The proved results of supercloseness and ultraconvergence were obtained under the requirement $u \in H^7(\Omega)$, which is too restrictive from a practical point of view.
- A fourth-order eigenvalue problem defined on an interval is considered in order to support the theory. A theory concerning two-dimensional polyrectangular domains cannot be illustrated by examples on an interval.
- Although the example in [8.10] is considered in a one-dimensional domain, very few finite elements are used. The finest triangulation has only 32 elements.

- If we still assume that the example is related to the theory under consideration, then this example refutes the obtained theoretical results for ultraconvergence. In [8.10], the authors have selected an example with known exact solutions satisfying $u_j \in H^7(\Omega)$, $\Omega = (0,1)$. We apply a formula for calculating an approximate convergence rate [1,2], taking into account that the exact eigenvalues are known. We use the two finer triangulations for the calculations we make. Thus, we obtain the following approximate convergence rates:

  $$\tilde{r}_1 = 2.58496, \tilde{r}_2 = 9.19319, \tilde{r}_3 = 2.81669$$

  for the three smallest approximate eigenvalues $\tilde{\lambda}_{h,j}$. The finite element method gives the most accurate results for the smallest eigenvalue. The obtained rates of convergence are far from the expected:

  $$\tilde{r}_j \approx 10, j = 1,2,3.$$

  The situation is similar with eigenfunctions. The approximate convergence rates for the eigenfunctions are:

  $$\tilde{\rho}_1 = 1.45251, \tilde{\rho}_2 = 4.61369, \tilde{\rho}_3 = 1.33770.$$

  According to the theory presented in [8.10], the approximate convergence rates of the eigenfunctions $\tilde{\rho}_j$ should approximate the number 5.
- If a theory cannot be applied to one-dimensional domains, then the same theory will have even less application to two-dimensional domains.

[1] Li H, Mazzucato A, Nistor V 2010 Analysis of the finite element method for transmission/mixed boundary value problems on general polygonal domains, Electronic Transactions on Numerical Analysis, 37, 41-69.
[2] H. Safdari, M. Rajabzadeh, M. Khalighi, LDG approximation of a nonlinear fractional convection-diffusion equation using B-spline basis functions, Applied Numerical Mathematics, Volume 171, 2022, Pages 45-57.

9. Conclusion

Overall, my opinion of the results presented by Assoc. Prof. Racheva is positive. I recommend that the scientific jury propose to the Scientific Council of the Faculty of Economics at the Technical University of Gabrovo to elect Assoc. Prof. DSc Milena Radoslavova Racheva to hold the academic position "professor" in the field of higher education 4. "Natural Sciences, Mathematics and Informatics", professional field 4.5 "Mathematics", major "Computational Mathematics".

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Gabrovo /Prof. Dr. Todor Todorov/